HYDRODYNAMIC ANALYSIS OF SHOCK ADIABATS OF HETEROGENEOUS MIXTURES OF SUBSTANCES

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We examine shock transformations in heterogeneous continua at high pressures, when each of the phases can be considered as a liquid (neglecting strength effects). We identify the variants of the state of complete interphase thermomechanical equilibrium and the temperature-frozen state immediately behind the shock. The compression of each of the phases in the mixture does not take place along the Hugoniot adiabat for the continuous material. The well-known anomalously high temperature rise during compression of materials with hollow pores [1, 2] is a particular case of such deviations.

In contrast with multicomponent media, the multiphase medium is composed of heterogeneous elementary particles whose state is determined by the parameters characteristic for continuum mechanics, namely the density, pressure, and so on; these parameters are related with one another in accordance with the equation of state of the corresponding continuous material of the phase. In this connection, when examining shock waves in media which are a mixture of condensed matter the problem arises of calculating the shock adiabat from the known equations of state for the phases (for example, the calculation of the shock adiabat of rocks from the given equations of state of the petrogenic materials), or the problem of finding the equation of state of the continuous material of one of the phases from the measured shock adiabat of the mixture and the known equations of state of the other phases.

1. For the analysis we use the model of two interpenetrating [3, 4] deformable continuous media. Considering only the hydrodynamic approach to the study of shock transformations, we simplify the model [5, 6], assuming that each of the condensed phases is a compressible liquid and that the phase pressures are equal. Then we have the equations of motion [5]

$$m\rho_1 \frac{d_1 u_i}{dt} = -m \frac{\partial p}{\partial x_i} + R_i, \quad (1-m)\rho_2 \frac{d_2 w_i}{dt} = -(1-m)\frac{\partial p}{\partial x_i} - R_i \tag{1.1}$$

the continuity equations for the phases

$$\frac{\partial \rho_1 m}{\partial t} + \frac{\partial \rho_1 m u_i}{\partial x_i} = 0, \qquad \frac{\partial \rho_2 (1-m)}{\partial t} + \frac{\partial \rho_2 (1-m) w_i}{\partial x_i} = 0 \tag{1.2}$$

and the total energy-balance equations [5, 6]

$$\begin{gathered}
\frac{\partial}{\partial t}\left\{\rho_1 m\left(\varepsilon_1 + \frac{u_i^2}{2}\right)\right\} + \frac{\partial}{\partial x_i}\left\{\rho_1 m\left(\varepsilon_1 + \frac{u_i^2}{2}\right)u_i - mpu_i\right\} + \frac{\delta W}{dt} + q = 0 \\
\frac{\partial}{\partial t}\left\{\rho_2 (1-m)\left(\varepsilon_2 + \frac{w_i^2}{2}\right)\right\} + \frac{\partial}{\partial x_i}\left\{\rho_2 (1-m)\left(\varepsilon_2 + \frac{w_i^2}{2}\right)w_i - (1-m)pw_i\right\} - \frac{\delta W}{dt} - q = 0 \\
\frac{\delta W}{dt} = p\frac{\partial m}{\partial t} + (1-\alpha)R_i u_i + \alpha R_i w_i
\end{gathered} \tag{1.3}$$

Here ρ_1 and ρ_2 are the densities; u_i and w_i are the phase velocities; m is the volumetric concentration of the first phase; R_i is the bulk interphase interaction force, depending on the difference of the phase velocities; ε_1 and ε_2 are the internal energies of the phases; q is the interphase heat flow. We neglect the heat flows in each of the phases.

Multiplying Eqs. (1.1) by u_i and w_i, respectively, we obtain the equations for the kinetic energies of the phases. Then, subtracting them from (1.3), we obtain the conservation equations for the internal energies of the phases:

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$$\rho_1 m\left\{\frac{d_1\varepsilon_1}{dt} + p\frac{d_1}{dt}\left(\frac{1}{\rho_1}\right)\right\} = \alpha R_i(w_i - u_i) + q$$
$$\rho_2(1-m)\left\{\frac{d_2\varepsilon_2}{dt} + p\frac{d_2}{dt}\left(\frac{1}{\rho_2}\right)\right\} = (1-\alpha)R_i(w_i - u_i) - q \tag{1.4}$$
$$\frac{d_1}{dt} = \frac{\partial}{\partial t} + u_i\frac{\partial}{\partial x_i}, \quad \frac{d_2}{dt} = \frac{\partial}{\partial t} + w_i\frac{\partial}{\partial x_i}$$

and the sense of the introduced coefficient α (see [7]) becomes clear: the work $R_i(w_i-u_i)$ of the bulk interphase interaction forces transforms into heat, with the part $(1-\alpha)$ in phase 1, and the part α in phase 2. We recall [5, 6] that if one of the phases (the second) is a solid, while the other is a liquid, then $\alpha = 1$.

We introduce the change of the average entropies s_i of the phases as the quotient of the average heat influx (to the particles of the phase in question per unit volume) and the average phase temperature

$$\rho_1 m\frac{d_1 s_1}{dt} = \frac{1}{T_1}\{\alpha R_i(w_i - u_i) + q\}$$
$$\rho_2(1-m)\frac{d_2 s_2}{dt} = \frac{1}{T_2}\{(1-\alpha)R_i(w_i - u_i) - q\} \tag{1.5}$$

The Gibbs relations for the phases is

$$\frac{d_1\varepsilon_1}{dt} + p\frac{d_1}{dt}\frac{1}{\rho_1} = T_1\frac{d_1 s_1}{dt}, \quad \frac{d_2\varepsilon_2}{dt} + p\frac{d_2}{dt}\frac{1}{\rho_2} = T_2\frac{d_2 s_2}{dt} \tag{1.6}$$

We assume hereafter that the functions $\varepsilon_i = \varepsilon_i(\rho_i, s_i)$, relating the parameters averaged in an elementary volume over the particles of the i-th phase, coincide with the relations characteristic for the continuous material of the corresponding phase. The closing relations are the kinetic relations for R_i and q which follow, for example, from the formalism of the thermodynamics of irreversible processes [6].

2. To analyze the shock transformations we examine, as usual [1], stationary one-dimensional solutions of (1.1)–(1.5) to the traveling-wave type, i.e., we seek solutions which depend on the single variable $\zeta = x - Ut$. Then we obtain the conservation integrals for the over-all momentum of the medium,

$$\rho_1 m(u-U)^2 + \rho_2(1-m)(w-U)^2 + p = Q \tag{2.1}$$

mass continuity for each of the phases

$$\rho_1 m(u-U) = M_1, \qquad \rho_2(1-m)(w-U) = M_2 \tag{2.2}$$

over-all total energy conservation

$$M_1\left(\varepsilon_1 + \frac{u^2}{2}\right) + M_2\left(\varepsilon_2 + \frac{w^2}{2}\right) + mpu + (1-m)pw = E \tag{2.3}$$

momentum- and energy-balance equations for one of the phases (the first, as an example)

$$\frac{d}{d\zeta}\{M_1 u + mp\} - p\frac{dm}{d\zeta} = +R \tag{2.4}$$

$$\frac{d}{d\zeta}\left\{M_1\left(\varepsilon_1 + \frac{u^2}{2}\right) + mpu\right\} - Up\frac{dm}{d\zeta} - R(\alpha w + (1-\alpha)u) + q = 0 \tag{2.5}$$

Multiplying (2.4) by U and subtracting from (2.5), we obtain

$$\frac{d}{d\zeta}\left\{M_1\left(\varepsilon_1 + \frac{u^2}{2}\right) + mp(u-U) - M_1 uU\right\} = -q + R\{\alpha w + (1-\alpha)u - U\} \tag{2.6}$$

We see from (2.4) and (2.6) that as $|\zeta| \to \infty$, i.e., far from the transitional flow regime, the conditions $R = 0$, $q = 0$ are satisfied, which means $u = w$, $T_1 = T_2$; in these distant regions the conditions of mechanical and thermodynamic interphase equilibrium are satisfied.

3. If in the transitional flow region, shock transition from one state into the other is realized, the integrals (2.1)–(2.3) will relate the flow parameters u_0, w_0, p_0, ... ahead of the shock front with those (u, w, and p) behind the front.

Then we obtain

$$M_1 u_* + M_2 w_* + p = M_1 u_{*0} + M_2 w_{*0} + p_0 \tag{3.1}$$

$$\rho_1 m u_* = \rho_{10} m_0 u_{*0} = M_1, \quad \rho_2(1-m)w_* = \rho_{20}(1-m_0)w_{*0} = M_2 \tag{3.2}$$

$$M_1(\varepsilon_1 + {}^1/_2 u_*{}^2) + M_2(\varepsilon_2 + {}^1/_2 w_*{}^2) + mpu_* + (1-m)pw_* \tag{3.3}$$
$$= M_1(\varepsilon_{10} + {}^1/_2 u_{*0}{}^2) + M_2(\varepsilon_{20} + {}^1/_2 w_{*0}{}^2) + m_0 p_0 u_{*0} + (1-m_0)p_0 w_{*0}$$

where $u_* = u - U$, $w_* = w - U$ are the phase-particle velocities relative to the shock wave front, which itself travels with velocity U.

Integrating (2.6) over the narrow interval $\zeta_0 - h$, $\zeta_0 + h$, which includes the shock front, and then passing to the limit as $h \to 0$, we obtain the following relation (under the assumption that only the derivatives can increase without bound in the interval, while the variables w and u themselves and also their functions q and R change stepwise, but that these changes are bounded):

$$M_1(\varepsilon_1 + {}^1/_2 u_*{}^2) + mpu_* = M_1(\varepsilon_{10} + {}^1/_2 u_{*0}{}^2) + m_0 p_0 u_{*0} \tag{3.4}$$

The closing relation must be the corollary from the equation of motion (2.4) of one of the phases, relating the motion parameters of this phase ahead of and behind the shock. It is also written in the form

$$u_* \frac{du_*}{d\zeta} + \frac{1}{\rho_1}\frac{dp}{d\zeta} = \frac{R}{m\rho_1} \tag{3.5}$$

Hence we see immediately that integration in the general case for $\rho_1 = \rho_1(p, T_1)$ requires preliminary expression from the indicated finite relations of the temperature T_1 in terms of the quantities U and p, and this involves analytic difficulties and in principle is possible only for a priori known equations of state of the phases. Only in the particular case of relatively small thermal part of the pressure, when $\rho_1 = \rho_1(p)$, can (3.5) be reduced to a simple balance relation [8, 9].

4. In the following we examine the case of such a large interphase interaction force R that the following condition is satisfied almost immediately behind the shock:

$$u \equiv w \tag{4.1}$$

For computational purposes we assume that (4.1) is satisfied immediately behind the shock front, i.e., it closes the balance relations (3.1)–(3.4). In this case the mass-balance equations (3.2) imply conservation of the phase mass concentrations in the flow,

$$\begin{aligned} x_1 &= \frac{m\rho_1}{m\rho_1 + (1-m)\rho_2} = \frac{M_1}{M_1 + M_2} = \text{const} \\ x_2 &= \frac{(1-m)\rho_2}{m\rho_1 + (1-m)\rho_2} = \frac{M_2}{M_1 + M_2} = \text{const}, \quad x_1 + x_2 = 1 \end{aligned} \tag{4.2}$$

We introduce the phase specific volumes v_i and the heterogeneous medium specific volume V (see also [10])

$$v_1 = \frac{1}{\rho_1}, \quad v_2 = \frac{1}{\rho_2}, \quad V = x_1 v_1 + x_2 v_2 = \frac{1}{m\rho_1 + (1-m)\rho_2} \tag{4.3}$$

Then (3.1) and (3.2) for $u_0 = w_0 = -U$ are written in the form

$$\frac{u_*{}^2}{V} + p = \frac{U^2}{V_0} + p_0, \qquad \frac{u_*}{V} = -\frac{U}{V_0} \tag{4.4}$$

and may be transformed to the usual form

$$U^2 = V_0{}^2 \frac{p - p_0}{V_0 - V}, \quad u_*{}^2 = V^2 \frac{p - p_0}{V_0 - V} \tag{4.5}$$

With the use of (4.4) the internal-energy increase of the medium (3.3) is written in the form

$$x_1(\varepsilon_1 - \varepsilon_{10}) + x_2(\varepsilon_2 - \varepsilon_{20}) = {}^1/_2 (p + p_0)(V_0 - V) \tag{4.6}$$

where $\varepsilon_1 = \varepsilon_1(v_1, T)$, $\varepsilon_2 = \varepsilon_2(v_2, T)$. We note the possibility of the transformation

$$\begin{aligned} V_0 - V &= V_0(1 - \rho_{10} m_0 v_1 - \rho_{20}(1-m_0)v_2) = V_0(m_0 - \rho_{10} m_0 v_1 \\ &\quad + 1 - m_0 - \rho_{20}(1-m_0)v_2) = x_1(v_{10} - v_1) + x_2(v_{20} - v_2) \end{aligned}$$

Therefore the balance relation (4.6) for the phase energies can also be written in the form

$$x_1\left\{\varepsilon_1 - \varepsilon_{10} - {}^1/_2\,(p + p_0)\,(v_{10} - v_1)\right\} + x_2\left\{\varepsilon_2 - \varepsilon_{20} - {}^1/_2\,(p + p_0)\,(v_{20} - v_2)\right\} = 0 \tag{4.7}$$

5. We first examine the situation in which heat exchange between the phases is so intense (q large) that we can consider the phase temperatures equal immediately behind the front,

$$T_1 = T_2 = T \tag{5.1}$$

Then (5.1) replaces (3.4). Let the internal energy of ε_i of the i-th phase be connected with the pressure p and volume v_i by the usual relation

$$\varepsilon_i = X_i\,(v_i) + c_i^v T_i = X_i\,(v_i) + \frac{v_i\,(p - p_i^x)}{\Gamma_i\,(v_i)} \tag{5.2}$$

Here $X_i(v_i)$ and p_i^x are the cold energy and pressure of the material of the i-th phase; c_i^v is its specific heat at constant volume; $\Gamma_i\,(v_i)$ is the Gruneisen coefficient [1]. Then (4.5) states that

$$\frac{v_1\,(p - p_1^x\,(v_1))}{\Gamma_1\,(v_1)\,c_1^v} = \frac{v_2\,(p - p_2^x\,(v_2))}{\Gamma_2\,(v_2)\,c_2^v} \tag{5.3}$$

and together with the equation

$$x_1\left\{X_1\,(v_1) + \frac{v_1\,(p - p_1^x)}{\Gamma_1\,(v_1)} - \frac{p\,(v_1^\circ - v_1)}{2}\right\} + x_2\left\{X_2\,(v_2) + \frac{v_2\,(p - p_2^x)}{\Gamma_2\,(v_2)} - \frac{p\,(v_2^\circ - v_2)}{2}\right\} = 0 \tag{5.4}$$

defines the degree of compression of the material of each of the phases behind the wave front. Here we have neglected the energy ε_0 and pressure p_0 as usual (for moderately strong shock waves [1]).

If the compression of the material of each of the phases were to take place along its shock adiabat, then (5.3) would be equivalent to the condition of vanishing of one of the braces in (5.4), which cannot occur in the general case.

6. A study of the thermodynamic properties of substances from observations of the passage of shock waves through a disperse mixture of the condensed substances was undertaken in [11, 12]. It was assumed that the state immediately behind the shock wave front is in mechanical equilibrium but is thermally frozen (adiabatic) (the dimensions of the particles of each of the phases are an order larger than the characteristic temperature-rise scales during the time of the shock compression process – about 10^{-6} sec).

These conditions correspond to (4.5), (4.7) together with (3.4), which can also be written in the form

$$\varepsilon_1 - \varepsilon_{10} + pv_1 - p_0 v_{10} = {}^1/_2\,(p - p_0)\,(V + V_0) \tag{6.1}$$

or ($x_1 = x$, $x_2 = 1 - x$)

$$\varepsilon_1 - \varepsilon_{10} = {}^1/_2\,(p + p_0)\,(V_0 - V) + (1 - x)\,\{p\,(v_2 - v_1) - p_0\,(v_{20} - v_{10})\} \tag{6.2}$$

If we can neglect the initial values ε_0, p_0, the shock-adiabat equations (4.7) and (6.2) take the form

$$x_1\varepsilon_1 + x_2\varepsilon_2 = {}^1/_2 p\,(V_0 - V) \tag{6.3}$$

$$\varepsilon_1 = {}^1/_2 p(V_0 - V) + (1 - x)p\,(v_2 - v_1) \tag{6.4}$$

Hence

$$\varepsilon_2 = {}^1/_2\,p(V_0 - V) + xp\,(v_1 - v_2) \tag{6.5}$$

The relations (6.4), (6.5) for shock compression of the phases can also be written in the form

$$\varepsilon_1 = {}^1/_2 p(v_{10} - v_1) + (1 - x){}^1/_2 p(v_{20} - v_{10} + v_2 - v_1) \tag{6.6}$$

$$\varepsilon_2 = {}^1/_2 p(v_{20} - v_2) + x{}^1/_2 p(v_{10} - v_{20} + v_1 - v_2) \tag{6.7}$$

Equations (6.3) and (6.4) show that the work which is expended on increasing the internal energy of the mixture of substances is distributed nonuniformly among the phases, and this distribution depends both on the mass content of the substances in the mixture and the magnitudes of the specific volumes of the phases in the compressed state [see (6.4) and (6.5)].

The second terms on the right-hand sides of (6.6) and (6.7) show how much the shock compression of the materials of the phases in the mixtures deviates (to the right or left on the pv-plane for the same p) from the shock compression curve for these of materials in the monolithic state (as a function of the magnitudes of the specific volumes of the phases). Here it is significant that the internal energy of one of the phases increases more and that of the other increases less than along the Hugoniot adiabats of the corresponding continua.

In experimental studies [13] it is possible to measure the shock wave velocity U and the mass velocity u_*, and the initial state of the medium is known, i.e., in the case in question x, v_{10}, and v_{20} are known. From (4.5) we can then calculate the pressure p and specific volume V of the mixture in the compressed state, i.e., we can plot the shock adiabat of the mixture. If we know one of the functions $\varepsilon_i(p, v_i)$, i.e., $p_i^x(v_i)$ and $\Gamma_i(v_i)$ [see, for example, (5.2)], then using the corresponding equation (6.4) or (6.5) for each of the pairs of values p, $V = xv_1 + (1-x)v_2$ we can calculate the quantities v_1 and v_2. Thereby we can plot the shock mixture adiabats for the materials of the phases. Combination of this adiabat with the remaining equation of the system (6.4), (6.5) leads to a differential equation of the type

$$f(X_i,\ p_i^x,\ \Gamma(v_i),\ p,\ v_1,\ v_2) = 0,\quad p_i^x = -dX_i/dv_i \tag{6.8}$$

Here the values of p, v_1, and v_2 are taken along the adiabat (for example, $p = p(v_1)$, $v_2 = v_2(v_1)$ are known functions of $v = v_1$).

If the relation $\Gamma(v_1)$ is known a priori, then it is sufficient to integrate along one of the adiabats (for a single value of the parameter x). However, if the function $\Gamma(v_1)$ is not known a priori, the mass content of the phases must be varied to find it.

Now let us examine the important (and already known [1, 2]) particular case in which the heterogeneous medium is represented by a porous material with hollow pores (the hollow pores constitute the second "phase" of the mixture). Then we have

$$\rho_{20} = \rho_2 = 0,\quad x_1 = x = 1,\quad x_2 = 1 - x = 0$$

$$x_2 v_{20} = \frac{1-m_0}{m_0\rho_{10}},\quad x_2 v_2 = \frac{1-m}{m\rho_1},\quad V_0 = \frac{1}{m_0\rho_{10}},\quad V = \frac{1}{m\rho_1} \tag{6.9}$$

which permits transforming (6.4) to a form which is valid for the porous state of the substance

$$\varepsilon_1 = \frac{1}{2}p\left(\frac{1}{m_0\rho_{10}} - \frac{1}{m\rho_1}\right) + p\frac{1-m}{m}\frac{1}{\rho_1} \tag{6.10}$$

Hence we see that if in the course of shock compression there is complete closure of the pores, i.e., $m = 1$, $V = v_1 = 1/\rho_1$, then exactly half the work performed in shock compression of the heterogeneous medium goes to increase the internal energy ε_1 of the first phase. The resulting anomalously high temperature rise of the originally porous material is widely used for measuring its equations of state at the points of the pv-plane above the shock adiabat of this substance in monolithic form [2].

However, if complete closure of the pores does not occur, then the equation shock adiabat (6.10) is somewhat more complex. In the latter case, however, the hydrodynamic approach may be insufficient (compare [14]).

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